



Size effect

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Abstract

This paper surveys the available results on the size effect on the nominal strength of structures — a fundamental problem of considerable importance to concrete structures, geotechnical structures, geomechanics, arctic ice engineering, composite materials, etc., with applications ranging from structural engineering to the design of ships and aircraft. The history of the ideas on the size effect is briefly outlined and recent research directions are emphasized. First, the classical statistical theory of size effect due to randomness of strength, completed by Weibull, is reviewed and its limitations pointed out. Subsequently, the energetic size effect, caused by stress redistributions due to large fractures, is discussed. Attention is then focused on the bridging between the theory of plasticity, which implies no size effect and is applicable for quasibrittle materials only on a sufficiently small scale, and the theory of linear elastic fracture mechanics, which exhibits the strongest possible deterministic size effect and is applicable for these materials on sufficiently large scales. The main ideas of the recently developed theory for the size effect in the bridging range are sketched. Only selected references to the vast amount of work that has recently been appearing in the literature are given. © 1999 Published by Elsevier Science Ltd. All rights reserved.

1. Introduction and classical history

The scaling, i.e., the change of response when the spatial dimensions are scaled up or down while the geometry and all other characteristics are preserved, is a quintessential problem of every physical theory. If the scaling is not understood, a viable theory does not exist. The scaling phenomena are particularly intricate in fluid mechanics. In that field, the scaling problems have been receiving major attention for more than a century. In solid mechanics, by contrast, the attention to the problem of scaling fluctuated and has been keen only in the very early and very recent history. This article will attempt a compact review of the history and the main results. A detailed exposition can be found in the comprehensive book by Bažant and Planas (1998) and extensive reviews in Bažant and Chen (1997), Bažant (1997a,b) and Bažant (1999).

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The question of size effect on the strength of ropes was discussed by Leonardo da Vinci (1500's), already a century before the concept of stress emerged in Galileo's work (Williams, 1957). Galileo (1638) rejected the size effect law proposed by Leonardo, consisting of inverse proportionality of the strength of rope to its length, and speculated on the size effect on the bones of large animals, calling their bulkiness the "weakness of the giants". Half a century later, Mariotte (1686) contributed, on the basis of his extensive experiments, a major idea which underlies the statistical theory of size effect. He observed that "a long rope and a short one always support the same weight unless that in a long rope there may happen to be some faulty place in which it will break sooner than in a shorter", which he called the principle of "inequality of matter whose absolute resistance is less in one place than in another".

Subsequently, not much happened until Griffith (1921) experimentally demonstrated that the nominal strength of glass fibers increases from 42,300 psi to 491,000 psi when the diameter is decreased from 0.0042 in. to 0.00013 in. He observed that "the weakness of isotropic solids... is due to the presence of discontinuities or flaws". "The effective strengths of technical materials could be increased 10 or 20 times at least if these flaws could be eliminated". This observation provided the physical basis of Marriotte's statistical idea. The mathematics of the statistical size effect, consisting in the weakest-link model for a chain and the extreme value statistics, emerged in the works of Tippett (1925), Peirce (1926), Fisher and Tippett (1928), Fréchet (1927) and von Mises (1936) (see also: Freudenthal, 1956, 1968, 1981; Freudenthal and Gumbell, 1956; Evans, 1978).

The principles of the statistical size effect were completed by Weibull (1939, 1949, 1951, 1956). He concluded that the tail distribution of low strength values of an extremely small probability cannot be represented by any of the previously known distributions and introduced a new statistical distribution that now bears his name. This distribution was later justified theoretically on the basis of some reasonable hypotheses about the statistical distribution and the role of microscopic flaws or microcracks (Freudenthal, 1968, 1981). Many refinements and extensions of Weibull theory have later been proposed and continued until today (Evans, 1978; Beremin, 1983; Ruggieri and Dodds, 1996; Xia and Shih, 1996; Lei et al., 1998; Kittl and Diaz, 1988; Kittl and Diaz, 1990, 1989; Zaitsev and Wittmann, 1974; Mihashi and Zaitsev, 1981; Wittmann and Zaitsev, 1981; Zech and Wittmann, 1977; Mihashi, 1983; Mihashi and Izumi, 1977; Carpinteri, 1986; Carpinteri, 1989).

Until the mid 1980's, it was generally believed that if a size effect is observed, it must be of statistical origin, described by Weibull theory. Consequently, mechanics generally paid no attention to size effect and believed it should be relegated to the statisticians [the subject was not even mentioned in Timoshenko's (1953) monumental *History of the Strength of Materials*].

The reason for the long ignorance of a non-statistical, mechanistic size effect is doubtless the fact that the classical well-established theories of elasticity with a strength limit, plasticity or any theory in which the material failure criterion is expressed in terms of stress and strain exhibit no size effect. In other words, the nominal strength of the structure does not depend on its size when geometrically similar structures are compared. The nominal strength is a parameter of the load having the dimension of stress. It is typically defined as the load divided by the square of the characteristic dimension of the structure, in the case of three-dimensional similarity, or the load divided by the characteristic dimension and the thickness, in the case of two dimensional similarity, although definitions such as the maximum stress in the structure, or the stress at any homologous points of the structure can serve equally well.

The linear elastic fracture mechanics (LEFM), in which all the fracture process is assumed to occur in one point, the crack tip, exhibits the strongest possible size effect, in which the nominal strength is inversely proportional to the square root of structure size. However, this is true only if the cracks are large and geometrically similar, and not if they are microscopic, having a length that is a material characteristic. In that case, which is typical of fatigue embrittled metals and ceramics, the effect of

cracks intervenes only through the macroscopic material strength and there is again no deterministic size effect.

2. Power scaling and Weibull theory

It is self-evident that the size effect is determined by some characteristic length in the mathematical formulation of the structural analysis problem. In the classical theories of failure such as plasticity, elasticity with the material strength limit, viscoelasticity, viscoplasticity, etc., no characteristic length is present. On the basis of this fact alone it is possible to prove that the scaling of any quantity in any physical system must be expressed by a power law of the size D of the system (for fluid mechanics, see Barenblatt, 1979; Sedov, 1959; and, for solids, Bažant, 1993; Bažant and Chen, 1997). For the case of the scaling of strength or any quantity with the dimension of stress, the exponent of the power law is zero, which, in solid mechanics, came to be called the case of no size effect.

In linear elastic fracture mechanics (LEFM), there is also no characteristic length present (the material strength is not a material parameter in LEFM). So the scaling must also be a power law. From the J -integral it is easy to show that the exponent of the power law for strength or any quantity of the dimension of stress must be $-1/2$. In the plot of the logarithm of nominal strength versus the logarithm of structure size, the scaling laws of plasticity and LEFM are represented by a horizontal straight line and a descending straight line of slope $-1/2$, respectively.

Weibull statistical theory, too, as it turns out, involves no characteristic length. Indeed, calculations confirm that the scaling law for the size effect on nominal strength is a power law, the exponent of which is $-n_d/m$, where m is a parameter of the Weibull statistical distribution called the Weibull modulus, and $n_d = 1, 2$ or 3 for uni-, two-, or three-dimensional geometric similarity. The fact that Weibull theory contains no characteristic length is an objection to the applications of this theory to quasibrittle materials for which a characteristic length obviously exists, being dictated by the size of material inhomogeneities or the characteristic size of the fracture process zone (FPZ).

To calculate the size dependence of the nominal strength of structure, it is convenient to introduce a size-independent stress measure called the Weibull stress. This formalism was proposed by Beremin (1983) and extended by Ruggieri and Dodds (1996) and Lei et al. (1998) to take into account the contributions from the elements of the plastic zone at fracture front calculated by finite elements.

In Weibull theory, the size effect on the nominal strength arises from the fact that the larger the structure the greater the probability to encounter in its volume a material element of a given critically small strength. The theory works well and its hypotheses are well justified when the failure occurs as soon as a macroscopic flaw becomes a propagating macroscopic crack. The underlying hypothesis is that the inception of macroscopic crack growth in one small element of the structure causes failure, which means that small representative volumes of the material in the structure interact in the same way as the links of a chain, that is, in series coupling. However, when a large FPZ or a long stable crack, or both, can develop before reaching the maximum load, the series coupling model of a chain, underlying the weakest link theory, becomes inapplicable and the statistical size effect gets overpowered by the effect of stress redistributions on the maximum loads with the inherent energy release (Bažant and Planas, 1998).

In the case of quasibrittle materials, which exhibit a non-negligible material length and typically grow a large FPZ and large cracks before the maximum load, applications of the classical Weibull theory are questionable for several reasons (Bažant and Planas, 1998):

1. The power law of size effect, as already mentioned, implies the absence of any characteristic length of the material or structure.
2. The stress redistributions, along with the consequent energy release caused by a large FPZ or stable

- crack growth before the maximum load, engender a deterministic size effect, which is ignored.
3. In Weibull theory, every structure, as can be shown, is mathematically equivalent to a uniaxially stressed bar of a variable cross section, which means that no information on the structure geometry and the failure mechanism is taken into account.
 4. The theory predicts rather different size effects for two- and three-dimensional scalings, which contradicts experimental evidence.
 5. The theory does not agree well with the recent test results for concrete structures and sea ice plates, which show that the doubly logarithmic size effect plot of nominal strength versus size is not a straight line but a descending curve whose slope is getting steeper as the size increases, approaching the LEFM slope $-1/2$.
 6. The classical Weibull theory ignores spatial correlations of the failure probabilities of neighboring material elements, while generalizations based on some hypotheses about load sharing have been purely phenomenological, not reflecting the nonlocal characteristics of damage evolution.

3. Quasibrittle size effect and its history

The hypostatic property of quasibrittle materials is that fracture propagation depends both on the fracture energy of the material, G_f , and material strength, σ_0 . This fact implies Irwin's (1958) characteristic length $l_0 = EG_f/\sigma_0^2$, which approximately characterizes the size of the FPZ ($E =$ Young's modulus). Thus the key to the quasibrittle size effect is a combination of the concept of strength or yield with fracture mechanics.

Application of LEFM to concrete was first considered by Kaplan (1961) but subsequent test results showed significant disagreements (Kesler et al., 1971; Leicester, 1969; Walsh, 1972; Walsh, 1976). Leicester (1969) conducted tests of geometrically similar notched beams of different sizes, fitted the results by a power-type size effect, and observed that the optimum exponent was greater than $-1/2$, the value required by LEFM. He tried to explain it by noting that the strength of the stress singularity for sharp notches of a finite angle is less than for sharp cracks. This explanation, however, is questionable because notches of a finite angle cannot propagate, and because the singular stress field of notches of a finite angle gives a zero energy flux into the notch tip. Besides, Leicester's power law for size effect implied nonexistence of a characteristic length. Based on more extensive tests of geometrically similar notched beams of different sizes, Walsh was the first to make the doubly logarithmic plot of nominal strength versus size and note that it appears to be transitional between plasticity and LEFM, although he did not attempt to make a mathematical analysis and obtain a formula.

A different type of quasibrittle size effect was brought to light by Hillerborg et al. (1976) (see also Petersson, 1981), who extended the models of Barenblatt (1959) and Dugdale (1960) to formulate the cohesive (or fictitious) crack model for concrete characterized by a softening stress-displacement law for the crack opening. By finite element analysis, they showed that the failures of plain concrete beams exhibit a deterministic size effect, which agrees with the test data on the modulus of rupture.

At the same time, stability analysis of postpeak strain softening damage revealed that its localization into a damage band engenders a deterministic size effect on the postpeak deflections and energy dissipation of structures (Bažant, 1976). The crack band was shown to play a role similar to a sizable FPZ. Approximate energy release analysis has led to a simple formula for the size effect law in quasibrittle structures with a large crack at maximum load (Bažant, 1984): $\sigma_N \sim (1 + \beta)^{-1/2}$ where $\beta = D/D_0 =$ relative size of structure characterizing its brittleness and $D_0 =$ constant.

Later, this simple law was derived on the basis of asymptotic energy release analysis and equivalent LEFM (Bažant and Kazemi, 1990; Bažant and Planas, 1998). This also led to expressing the geometry

effect on the coefficients of this law through the LEFM energy release functions. Measurement of the size effect on the maximum load of notched specimens were shown to provide a simple means for determining the fracture energy and the effective length of FPZ, as well as the R-curve, and were embodied in a standard RILEM Recommendation (1990). The aforementioned size effect law was shown to agree with the test data of Walsh (1972) as well as with many subsequent test data of a much broader range, including not only concrete and mortar but also rocks, fibre-polymer composites, tough ceramics, sea ice and wood. The law was shown to also closely match the finite element results of the simple crack band model (Bažant and Oh, 1983), used in commercial codes (e.g. DIANA, SBETA; Cervenka and Pukl, 1994), as well as the cohesive crack model (Bažant and Planas, 1998).

The late 1980's saw a great surge of interest in the quasibrittle size effect and many researchers made valuable contributions (Planas and Elices, 1988, 1993, 1989; Petersson, 1981; Carpinteri, 1986; to name but a few). Recently, the problem of size effect in concrete structures has become a major theme at conferences (Bažant, 1992a, 1992b; Mihashi et al., 1994; Wittmann, 1995; Mihashi and Rokugo, 1998; Bažant and Rajapakse, 1999).

4. Recent studies of energetic size effect in quasibrittle structures

The source of the energetic size effect, briefly stated, is a mismatch between the size dependence of the energy release rate and the rate of energy consumption by fracture. A significant part of the former increases as the square of the structure size, while the latter increases in proportion. Therefore, the nominal stress must decrease to reduce the energy release rate of structure so as to achieve a match.

Two simple kinds of size effect may be distinguished:

1. the size effect in structures with notches or large cracks at the maximum load, and
2. the size effect when the failure occurs at the initiation of fracture from a smooth surface.

The former is typical of reinforced concrete structures in which the reinforcement makes possible stable growth of large cracks before the maximum load, and it also occurs in situations in which there is a large compressive stress parallel to the crack (e.g., the fracture of dams). The latter occurs when the maximum load in a material with a large FPZ is reached at fracture initiation from the surface (e.g., the modulus of rupture test).

A strong compressive stress with insufficient lateral confining pressure produces damage in the form of axial splitting cracks. This damage localizes into a band which can propagate either laterally or axially.

In the case of axial propagation, energy gets released only from the band and not from the rest of the structure. Because this energy release is proportional to the length of the band, there is no size effect.

In the case of lateral propagation, the stress in the zones on the sides of the compression damage band gets reduced, which causes an energy release that grows in proportion to the square of the structure size, while the energy dissipated in the band grows linearly with the size. Similar to tensile fracture, the mismatch of energy release rates inevitably produces a size effect. Because of the size effect, the failure by lateral propagation must prevail for sufficiently large sizes. Under the assumption that the spacing of the axial splitting cracks in the propagating band is independent of the structure size, a similar size effect law as for tensile fracture is obtained, except for an additive constant. However, minimization of the strength indicates that the crack spacing should increase with the structure size, in which case the analysis leads to a slightly different size effect formula which approaches in the doubly logarithmic plot an asymptote of slope $-2/5$ rather than $-1/2$. This formula was applied to describe the size effect on the breakout of boreholes in rock (Bažant and Planas, 1998), revealed by tests of Carter (1992) and Carter et al. (1992).

A particular type of compression failure is observed in fiber composites with unidirectional reinforcement (Rosen, 1965; Argon, 1972; Budiansky, 1983; Budiansky et al., 1997). It involves transverse propagation of a kink band in which the fibers undergo microbuckling. Axial shear-splitting cracks develop between the fibers and the axial normal stress transmitted across the kink band gets gradually decreased during microbuckling and probably approaches some finite residual value. Recent size effect tests of geometrically similar PEEK-carbon fiber specimens revealed the existence of a significant size effect (Bažant et al., 1999). Analysis of the energy release during the propagation of the kink band, coupled with plasticity type analysis of the residual carrying capacity, led to a size effect formula that is similar to that for compression fracture of concrete. This formula matches the test results reasonably well.

The size effect is important for the failure of floating sea ice plates in the Arctic. One problem that has been extensively investigated is the capacity of ice plate to carry concentrated vertical loads or its resistance to an object trying to penetrate the plate from below. Aside from the need to analyze fracture in a plate on elastic foundation, a particular difficulty in this problem is the finding that the radial cracks emanating in a star pattern from the loaded area reach only through a part of the thickness, the depth profile of these vertically growing cracks being quite variable. The problem has recently been analyzed in great detail numerically, and the calculations have again revealed a size effect that represents a bridging between plasticity, applicable for ice plates less than about 0.2 m thick, to LEFM, applicable for thicknesses over about 0.5 m (Bažant and Kim, 1998a, 1998b). The results of this analysis agree reasonably well with experiments. The size effect is also manifested acoustically (Li and Bažant, 1998).

Another type of size effect in ice which has proven to be easy to analyze is the effect of ice thickness on the propagation of long thermal fracture caused by rapid cooling in the Arctic. These fractures can run for tens of kilometers and, curiously, are not bypassing the areas of thick ice but pass straight through them. An energy release analysis of the size effect showed that the critical temperature drop that causes propagation of such thermal cracks is inversely proportional to the $-3/8^{\text{th}}$ power of ice thickness. It came as a surprise that the exponent was not $-1/2$, however, it was found that this apparent anomaly is caused by the fact that the flexural wavelength of the floating ice plate, governing the decay of the unloading bending moments away from the fracture, is proportional not to the ice thickness but to its $4/3^{\text{rd}}$ power.

Recent experiments have further demonstrated that fiber composites, e.g. of carbon-epoxy type, exhibit a strong size effect, and that the size effect is reasonably well described by the aforementioned size effect law for quasibrittle tensile fracture (Bažant et al., 1996). In the case of such composites, an additional difficulty is that the energy release functions of LEFM must be calculated taking into account the orthotropy of the material. Demonstration of the existence of the size effect implies that that it ought to be taken into account in the design of large load-bearing fuselage panels, hulls of large ships, ship decks, bulkheads, stacks, masts etc. The statistical size effect of course also occurs in composites (e.g. Jackson et al., 1992).

Important fracture experiments on sea ice, and size effect tests of by far the broadest range ever conducted, have been reported by Dempsey et al. (1995) (also Mulmule et al., 1995). They deal with the horizontal propagation of full-through cracks (induced by horizontal forces of flat jacks inserted into a vertical notch) in floating notched fracture specimens of sizes from 0.5 m to 80 m. Good agreement with the aforementioned size effect law has been found. Extrapolation up to sizes of several kilometers was found to agree with the measurements of horizontal forces exerted by a moving ice floe on an oil platform in the Arctic, while previous predictions based on the laboratory strength of sea ice were an order of magnitude higher. In detail, see the article by Dempsey et al. (1999) in this volume.

In view of the energetic mechanism, it is not surprising that the size effect also afflicts the static fatigue crack growth in quasibrittle materials such as concrete. The Paris-Erdogan law for static crack

growth needs to be corrected with a factor based on the aforementioned size effect law. Such an extended formula has been shown to agree closely with test results.

Calculation of the size effect in structures with the cohesive crack model is considerably more difficult and is normally carried out by finite elements. However, it has recently been found that the maximum loads for structures of various sizes can be calculated directly, without integrating the load-deflection history (provided that no unloading occurs in the cohesive crack during the loading process). The analysis is reduced to one homogeneous Fredholm integral equation whose eigenvalue is the structure size for which a given relative fracture length leads to a maximum load (Li and Bažant, 1997). The maximum load may then be evaluated from the eigenvector as a ratio of two integrals. In this manner, a parametric description of the size effect curve for the cohesive crack model is obtained.

The capability to correctly reproduce the size effect is an important check on the validity of any computational model for a quasibrittle structure. To simulate the quasibrittle size effect transitional between plasticity and LEFM, nonlocal material models must be used for the description of softening damage in finite element programs. These can consist either of an integral-type nonlocal formulation or its approximation by a second-order gradient model. As a simple approach to simulate the size effect, the crack band model, in which the size of the elements in the crack band is considered to be a material property, may be employed (Bažant and Planas, 1998).

The energetic theory of the quasibrittle size effect is strictly deterministic, yet the material properties are certainly random. Does this randomness have any effect? It does, but on the mean only little. Studies of this question have indicated that, because of stress redistributions and concentrations due to fracture, the mean statistical size effect is wiped out. The reason is that the FPZ is approximately of the same size for structures of various sizes, and the major contribution to the Weibull probability integral over the structure volume comes from the FPZ. The randomness of the material nevertheless intervenes in the mean asymptotic behavior. For the size effect applicable to notched structures or structures with large cracks, there is a transition of the quasibrittle size effect to the Weibull type size effect for very small sizes for which the FPZ occupies essentially the entire structure. On the other hand, for structures failing at the initiation of macroscopic fracture, there is a transition to Weibull type size effect for structures of very large sizes. These transitions to probabilistic behavior, however, appear to take place, at least for concrete structures, outside the size range that is of practical interest (Bažant and Chen, 1997).

5. Other types of size effect and the fractal hypothesis

When the material exhibits time-dependent behavior such as viscoelasticity or viscoplasticity (creep), a different type of size effect, varying with time, is engendered. The reason is that the presence of viscosity in the material model implies a characteristic length of the material (material viscosity divided by wave velocity and mass density), as well as a characteristic time (the time a wave travels the characteristic length). The characteristic length poses a limit on the localization of damage within a fixed time interval and thus may produce what looks as a quasibrittle size effect bridging plasticity and LEFM. There is a difference, however. The localization limiting properties as well as the size effect engendered by material viscosity exist only within a certain limited range of loading rates and durations of loading. When this range is exceeded by a factor of 10 or more, these properties disappear. On the other hand, various quasibrittle materials, for example concrete, exhibit size effect and damage bands of finite thickness over an extremely broad range of delay times (load durations) or loading rates, spanning over about ten orders of magnitude. Such behavior cannot be captured by viscosity, and the energetic size effect discussed before is the proper approach.

Considerable interest and polemics have recently been generated by the idea that the physical origin

of the size effect observed in concrete structures might be the partly fractal nature of the crack surfaces and the distribution of microcracks in concrete (e.g., Carpinteri, 1994; Carpinteri et al., 1994; Carpinteri and Chiaia, 1995). Based on strictly geometric arguments, these authors proposed what they called the “multi-fractal scaling law” (MFSL), to be applied to the size effect in failures occurring at fracture initiation from a smooth surface. Four objections to this idea have, however, been raised (Bažant, 1997b):

1. A mechanical energy-based analysis (of either invasive or lacunar fractals) predicts a size effect trend that differs from MFSL and also disagrees with test data.
2. The fractal nature of the final fracture surface cannot matter because typically about 99% of energy is dissipated not on the final fracture surface but by microcracks and frictional slips in the FPZ at points lying away from that surface.
3. The fractal theory does not predict how the coefficients of MFSL depend on the structural geometry, which would greatly reduce the usefulness of MFSL for the design of structures.
4. The same formula as MFSL has been logically derived from (nonfractal) fracture mechanics, by asymptotic expansion of the energy release function of LEFM near the surface (Bažant, 1998).

Unlike fractality, the fracture explanation of the MFSL formula has the virtue that the geometry dependence of the coefficients of the MFSL formula can be readily determined, using LEFM.

In addition to the statistical, energetic quasibrittle and viscous size effects, there are three other types of size effect influencing the nominal strength of structures:

1. The boundary layer effect, arising from material heterogeneity, namely the fact that near the boundary the microstructure of heterogeneous material is different because:
 - (a) the aggregates or other inhomogeneities cannot protrude through the surface, and also
 - (b) because the Poisson effect causes the statistical microstress distributions to be different than those in the interior.
2. The existence of a three-dimensional stress singularity at the intersection of crack edge with a surface, which is also engendered by the Poisson effect (Bažant and Planas, 1998; Sec. 1.3), and causes the portion of the FPZ near the surface to behave differently from the interior portion.
3. Further time-dependent size effects caused by diffusion phenomena such as the transport of heat or of moisture and chemical agents in porous solids, which is manifested, e.g., through the effect of structure size on the shrinkage, drying creep and cracking of concrete, due to the size dependence of the drying half-times (Planas and Elices, 1993).

6. Closing comments on research trends

Although a large progress has been achieved in the understanding of the size effect in solids, and quasibrittle materials in particular, much further research is needed. Since the physical basis of size effect and the characteristic material length resides in the nonlocal behavior of the material, the modeling of the physical processes in the microstructure which endow the material with nonlocal characteristics needs to be greatly improved, and the bridging between the microscale of a heterogeneous material with distributed microcracks and frictional slips on one hand, and the macroscopic continuum description on the other hand, needs to be mastered.

Much enlightenment, though, can be gained from finite element and discrete element modeling of the microstructure, especially if it can be extended to a truly three-dimensional modeling. Analytical descriptions of the connection between the microstructural phenomena and the macroscopic continuum

are nevertheless extremely important, for only those descriptions can provide true understanding. Only they can capture the main aspects of the behavior and obviate the unimportant microscopic phenomena that cancel each other on the macroscale.

While the existing nonlocal models are essentially phenomenological, relying on assumed scalar spatial averaging of damage, the damage interactions are in reality oriented and tensorial. The effect of each microcrack or slip on another microcrack or slip, which is the source of nonlocality, decays with distance by rules dictated by elasticity. If all such behavior is taken into account, the conclusion is that the nonlocal spatial interactions must be tensorial and directional, that is, the kernel of the spatial integral providing the macroscopic smoothing should be based on the stress fields of microcracks and frictional slips and their long-range decay. Statistical arguments on the microstructural level should of course be also introduced to obtain the macroscopic average behavior as well as its variance.

In general, the statistical treatment of the size effect needs to be greatly improved with respect to localization (e.g. Bažant and Cedolin, 1991), and a full marriage of Weibull statistical theory with the energetic quasibrittle size effects in the interior and in the boundary layer of structures needs to be achieved. Simple design formulae for various design situations, for example the diagonal shear and torsional failures of reinforced concrete beams or punching shear failures of slabs (e.g. Reinhardt 1981; Walraven and Lehwalter, 1994; Bažant and Planas, 1993), as well as the tensile and compressive fractures of load-bearing fiber composite structures such as ship hulls, bulkheads and decks or load-bearing fuselage elements, need to be developed.

Finally, some segments of the engineering community need to be educated in the concepts of fracture mechanics, including the size effect aspects, in order to be willing to accept new improved design procedures, e.g., for concrete structures or fiber composites.

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